B. Sc. Part II 4th Paper

DYNAMICS

Q. A particle moves in the curve $y = a \log \sec \frac{x}{a}$ in such away that the tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as square of the radius of curvature.

Soln Given that tangent to the curve rotates uniformly.

$$\Rightarrow \frac{d\psi}{dt} = \text{constant} = k \text{ (suppose)} \quad \text{--- (1)}$$

The curve given is

$$y = a \log \sec \frac{x}{a} \quad \text{--- (2)}$$

$$\Rightarrow \frac{dy}{dx} = \cancel{a} \cdot \frac{1}{\cancel{\sec \frac{x}{a}}} \cdot \cancel{\sec \frac{x}{a}} \cdot \tan \frac{x}{a} \cdot \frac{1}{\cancel{a}}$$

$$\Rightarrow \frac{dy}{dx} = \tan \frac{x}{a} \quad \text{--- (3)}$$

$$\because \tan \psi = \frac{dy}{dx} \Rightarrow \tan \psi = \tan \frac{x}{a}$$

$$\Rightarrow \psi = \frac{x}{a} \Rightarrow \frac{d\psi}{dx} = \frac{1}{a} \quad \text{--- (4)}$$

From (1), $\frac{d\psi}{dt} = k \Rightarrow \frac{d\psi}{dx} \cdot \frac{dx}{dt} = \frac{1}{a} \cdot \frac{dx}{dt} = k$

$$\Rightarrow \frac{dx}{dt} = ak \quad (5) \Rightarrow \frac{d^2x}{dt^2} = 0 \quad \text{--- (6)}$$

Now, $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \tan \frac{x}{a} \cdot ak$ [From (3),(5)]

$$\Rightarrow \frac{dy}{dt} = ak \tan\frac{x}{a}$$

$$\Rightarrow \frac{d^2y}{dt^2} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dx}\left(ak\tan\frac{x}{a}\right)\cdot\frac{dx}{dt}$$

$$= ak \cdot \sec^2\frac{x}{a}\cdot\frac{1}{\cancel{a}}\cdot\cancel{a}k$$

$$\Rightarrow \frac{d^2y}{dt^2} = ak^2\sec^2\frac{x}{a} \quad \text{--- (7)}$$

Now, acceleration $(f) = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2}$ [From (6) & (7)]

$$= \sqrt{0 + \left(ak^2\sec^2\frac{x}{a}\right)^2}$$

$$\Rightarrow \text{acceleration, } f = ak^2\sec^2\frac{x}{a} \quad \text{--- (8)}$$

Now, $\rho = \dfrac{(1+y'^2)^{3/2}}{y''} = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$

$$= \frac{\left(1+\tan^2\frac{x}{a}\right)^{3/2}}{\frac{d}{dx}\left(\frac{dy}{dx}\right)} \quad \left[\because \frac{dy}{dx} = \tan\frac{x}{a}\right]$$

$$= \frac{\left(\sec^2\frac{x}{a}\right)^{3/2}}{\frac{d}{dx}\left(\tan\frac{x}{a}\right)} = \frac{\sec^3\frac{x}{a}}{\sec^2\frac{x}{a}\cdot\frac{1}{a}} = a\sec\frac{x}{a}$$

$$\Rightarrow \rho^2 = a^2\sec^2\frac{x}{a} \quad \text{--- (9)}$$

$\therefore$ from (8), $f = ak^2\sec^2\frac{x}{a} = \dfrac{k^2a^2\sec^2\frac{x}{a}}{a}$

$$= \frac{k^2\rho^2}{a} = \frac{\text{Const} \times \rho^2}{\text{constant}} = \text{Const} \times \rho^2$$

$\Rightarrow f \propto \rho^2$ proved